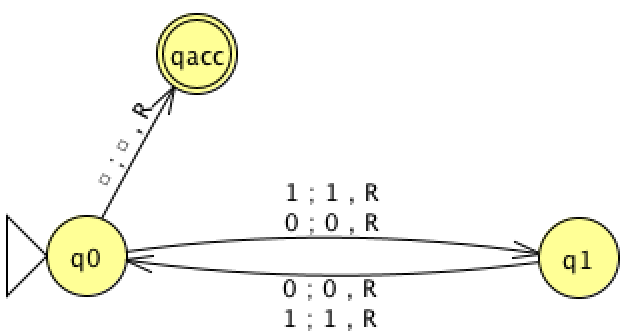
Week 8 Monday Review Quiz

Q1 ATM: examples

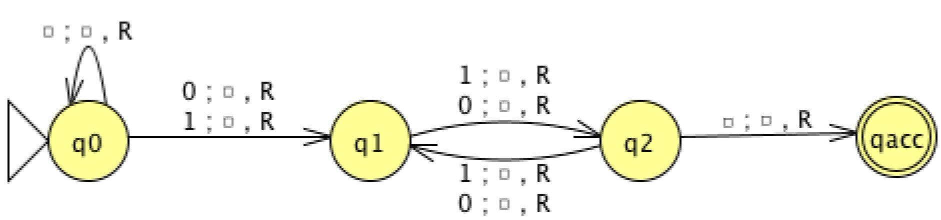
2 Points

Consider the following Turing machines over the alphabet $\{0, 1\}$, whose state diagrams are below.

M1



M2



Select all and only true statements below.

- ☐ $\langle M1 \rangle \in A_{TM}$
- ☐ $\langle M1 \rangle \in \overline{A_{TM}}$
- ☐ $\langle M2, \varepsilon \rangle \in A_{TM}$
- ☐ $\langle M2, 0 \rangle \in A_{TM}$
- ☐ $\langle M2, 00 \rangle \in A_{TM}$

Save Answer

Q2 ATM: "difficulty"

2 Points

Select all and only true statements below.

- [ ] A_{TM} is regular
- [ ] A_{TM} is context-free
- [ ] A_{TM} is Turing-decidable
- [ ] A_{TM} is Turing-recognizable
- [ ] A_{TM} is nonregular
- [ ] A_{TM} is non-context-free
- [ ] A_{TM} is undecidable
- [ ] A_{TM} is unrecognizable

Save Answer

Q3 Diagonalization proof

2 Points

What are the roles of the Turing machines R_{ATM}, M_{ATM} and D in the proofs that A_{TM} is recognizable but not decidable?

R_{ATM} and M_{ATM} both try to decide A_{TM} so they're not real Turing machines.

R_{ATM} is a well-defined Turing machine (but not a decider) that recognizes A_{TM}. M_{ATM} and D are deciders that could be built if we assume towards a contradiction that A_{TM} is decidable.

Save Answer

Q4 Closure

1 Point

Select all and only true statements below.

- [ ] The class of regular languages is closed under complementation
- [ ] The class of context-free languages is closed under complementation
- [ ] The class of decidable languages is closed under complementation
- [ ] The class of recognizable languages is closed under complementation
- [ ] The class of undecidable languages is closed under complementation
- [ ] The class of unrecognizable languages is closed under complementation

Save Answer

Q5 Co-recognizable

3 Points

A language L over an alphabet Σ is called co-recognizable if its complement, defined as $\Sigma^* \setminus L = \{x \in \Sigma^* \mid x \notin L\}$, is Turing-recognizable

Theorem 4.22 in the book says that a language is decidable if and only if it is both recognizable and co-recognizable.

Select all and only the sets below that are co-recognizable.

- [ ] A_{DFA}
- [ ] A_{NFA}
- [ ] A_{REX}
- [ ] A_{PDA}
- [ ] A_{TM}

Save Answer

Week 8 Wednesday Review Quiz

Q1 Computable functions

3 Points

Recall that a function $f : \Sigma^* \to \Sigma^*$ is computable means that there is some Turing machine M such that, on every input w, the computation of M on w halts with just $f(w)$ on its tape.

Q1.1 (a)

2 Points

Select all and only the functions below that are computable.

- [ ] The function $f_1 : \{0,1\}^* \to \{0,1\}^*$ such that $f_1(\varepsilon) = 001$ and $f_1(x) = \varepsilon$ for all $x \neq \varepsilon$.
- [ ] The function $f_2 : \{0,1\}^* \to \{0,1\}^*$ such that $f_2(x) = 0x0$ (i.e. the concatenation of 0 with x followed by a 0).

Save Answer

Q1.2 (b)
1 Point

True or False: The function

$$f_3(x) = \begin{cases} 0 & \text{if it will rain on campus during the CSE 105 final} \\ 1 & \text{otherwise} \end{cases}$$

is a computable function with domain Σ^* and codomain Σ^*

- ○ False, because the function f_3 is not well-defined.
- ○ False, because the function f_3 cannot be computed by any Turing machine.
- ○ True, because the function f_3 is a constant function (even though we may not know right now which constant value it outputs)

Save Answer

Q2 Mapping reduction
7 Points

Recall that mapping reduction is defined in section 5.3: The problem A mapping reduces to B means there is a computable function $f : \Sigma^* \to \Sigma^*$ such that for all $x \in \Sigma^*$, $x \in A$ iff $f(x) \in B$.
A computable function that makes the iff true is said to witness the mapping reduction from A to B.

Fix $\Sigma = \{0, 1\}$ throughout this question.

Q2.1
1 Point

Consider the statement: $\emptyset \leq_m \emptyset$ is witnessed by the computable function $id : \Sigma^* \to \Sigma^*$ given by $id(x) = x$ for all x.

- ○ This is a true statement.
- ○ The mapping reduction relationship is true but the given function does not witness this mapping reduction.
- ○ The mapping reduction relationship is not true.

Save Answer

Q2.2

1 Point

Consider the statement: $\Sigma^* \leq_m \Sigma^*$ is witnessed by the computable function $id : \Sigma^* \to \Sigma^*$ given by $id(x) = x$ for all x.

- ◯ This is a true statement.
- ◯ The mapping reduction relationship is true but the given function does not witness this mapping reduction.
- ◯ The mapping reduction relationship is not true.

Save Answer

Q2.3

1 Point

Consider the statement: $\Sigma^* \leq_m \emptyset$ is witnessed by the computable function $id : \Sigma^* \to \Sigma^*$ given by $id(x) = x$ for all x.

- ◯ This is a true statement.
- ◯ The mapping reduction relationship is true but the given function does not witness this mapping reduction.
- ◯ The mapping reduction relationship is not true.

Save Answer

Q2.4

1 Point

Consider the statement: $\{0, 1\} \leq_m \{00, 10\}$ is witnessed by the computable function $id : \Sigma^* \to \Sigma^*$ given by $id(x) = x$ for all x.

- ◯ This is a true statement.
- ◯ The mapping reduction relationship is true but the given function does not witness this mapping reduction.
- ◯ The mapping reduction relationship is not true.

Save Answer

Q2.5

1 Point

Consider the statement: $\{0, 1\} \leq_m \{00, 10\}$ is witnessed by the computable function $g : \Sigma^* \to \Sigma^*$ given by $g(x) = x0$ for all x.

- ◯ This is a true statement.
- ◯ The mapping reduction relationship is true but the given function does not witness this mapping reduction.
- ◯ The mapping reduction relationship is not true.

Save Answer

Q2.6

1 Point

Consider the statement: $\{00, 10\} \leq_m \{0, 1\}$ is witnessed by the computable function $g : \Sigma^* \to \Sigma^*$ given by $g(x) = x0$ for all x.

- ◯ This is a true statement.
- ◯ The mapping reduction relationship is true but the given function does not witness this mapping reduction.
- ◯ The mapping reduction relationship is not true.

Save Answer

Q2.7

1 Point

Consider the statement: $\{00, 10\} \leq_m \{0, 1\}$ is witnessed by the computable function $h : \Sigma^* \to \Sigma^*$ given by $h(x) = \begin{cases} x & \text{if } x = \varepsilon \\ \text{leftmost character of } x & \text{otherwise} \end{cases}$

- ◯ This is a true statement.
- ◯ The mapping reduction relationship is true but the given function does not witness this mapping reduction.
- ◯ The mapping reduction relationship is not true.

Save Answer

Week 8 Friday Review Quiz

Q1 Mapping reduction identities

5 Points

Fix $\Sigma = \{0, 1\}$ for this question.

Q1.1

2 Points

Select all and only the true statements.

- [ ] For languages A, B if A mapping reduces to B and B mapping reduces to A then $A = B$.
- [ ] For languages A, B, C if A mapping reduces to B and B mapping reduces to C then A mapping reduces to C.
- [ ] For languages A, B if A mapping reduces to B then B mapping reduces to A.

Save Answer

Q1.2

1 Point

True or false? "Every language mapping reduces to its complement."

- ○ True
- ○ False

Save Answer

Q1.3
1 Point

True or false? "Every decidable language mapping reduces to $\emptyset$."

- ○ True
- ○ False

Save Answer

Q1.4
1 Point

True or false? "Σ^* mapping reduces to every nonempty language over Σ"

- ○ True
- ○ False

Save Answer

Q2 Computable functions for mapping reductions
5 Points

Fix $\Sigma = \{0, 1\}$ and define $const_{out} \in \Sigma^*$ to be a string constant that is not the code of any pair of the form $\langle M, w \rangle$, where M is a Turing machine and w is a string.

Consider the computable function defined by the
high-level description of the TM computing it:

$F =$ "On input x:
1. If $x \neq \langle M, w \rangle$ for any Turing machine M and string w, output $const_{out}$.
2. Otherwise, let M be the Turing machine and w the string such that $x = \langle M, w \rangle$.
3. Define the Turing machine M' as :
   "On input y,
      1. Run M on y^R. If it accepts, accept. If it rejects, reject."
4. Output $\langle M', w^R \rangle$."

Q2.1

1 Point

True or False? "For all strings x, if $x \in A_{TM}$ then $F(x) \in HALT_{TM}$"

- True
- False

Save Answer

Q2.2

1 Point

True or False? "For all strings x, if $F(x) \in HALT_{TM}$ then $x \in A_{TM}$"

- True
- False

Save Answer

Q2.3

1 Point

True or False? "For all strings x, if $x \in HALT_{TM}$ then $F(x) \in A_{TM}$"

- True
- False

Save Answer

Q2.4

1 Point

True or False? For all strings x, if $F(x) \in A_{TM}$ then $x \in HALT_{TM}$

- True
- False

Save Answer

Q2.5
1 Point

Does $F(x)$ witness either of the the mapping reduction $HALT_{TM} \leq_m A_{TM}$ or $A_{TM} \leq_m HALT_{TM}$?

- ○ Yes, it witnesses both of these mapping reduction.
- ○ Yes, it witnesses $HALT_{TM} \leq_m A_{TM}$ (but not the other)
- ○ Yes, it witnesses $A_{TM} \leq_m HALT_{TM}$ (but not the other)
- ○ No, it does not witness either of these mapping reductions, but there are other computable functions that do.
- ○ No, it does not witness either of these mapping reductions, and at least one of these mapping reductions does not hold.

Save Answer

Q3 Feedback
0 Points

Any feedback about this week's material or comments you'd like to share? (Optional; not for credit)

Save Answer

Save All Answers

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