Week8 monday

Theorem: A_{TM} is not Turing-decidable.

Proof: Suppose **towards a contradiction** that there is a Turing machine that decides A_{TM}. We call this presumed machine M_{ATM}.

By assumption, for every Turing machine M and every string w

- If $w \in L(M)$, then the computation of M_{ATM} on $\langle M, w \rangle$ ______________________________
- If $w \notin L(M)$, then the computation of M_{ATM} on $\langle M, w \rangle$ ______________________________

Define a **new** Turing machine using the high-level description:

$D =$ " On input $\langle M \rangle$, where M is a Turing machine:

1. Run M_{ATM} on $\langle M, \langle M \rangle \rangle$.
2. If M_{ATM} accepts, reject; if M_{ATM} rejects, accept."

Is D a Turing machine?

Is D a decider?

What is the result of the computation of D on $\langle D \rangle$?

Theorem (Sipser Theorem 4.22): A language is Turing-decidable if and only if both it and its complement are Turing-recognizable.

Proof, first direction: Suppose language L is Turing-decidable. WTS that both it and its complement are Turing-recognizable.

Proof, second direction: Suppose language L is Turing-recognizable, and so is its complement. WTS that L is Turing-decidable.

Give an example of a **decidable** set:

Give an example of a **recognizable undecidable** set:

Give an example of an **unrecognizable** set:

True or **False**: The class of Turing-decidable languages is closed under complementation?

Definition: A language L over an alphabet Σ is called **co-recognizable** if its complement, defined as $\Sigma^* \setminus L = \{x \in \Sigma^* \mid x \notin L\}$, is Turing-recognizable.

Notation: The complement of a set X is denoted with a superscript c, X^c, or an overline, $\overline{X}$.