Week 10 Discussion CSE 105 Spring 2023

Agenda : Review session for finals

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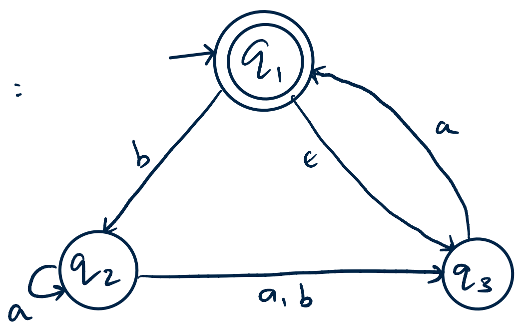
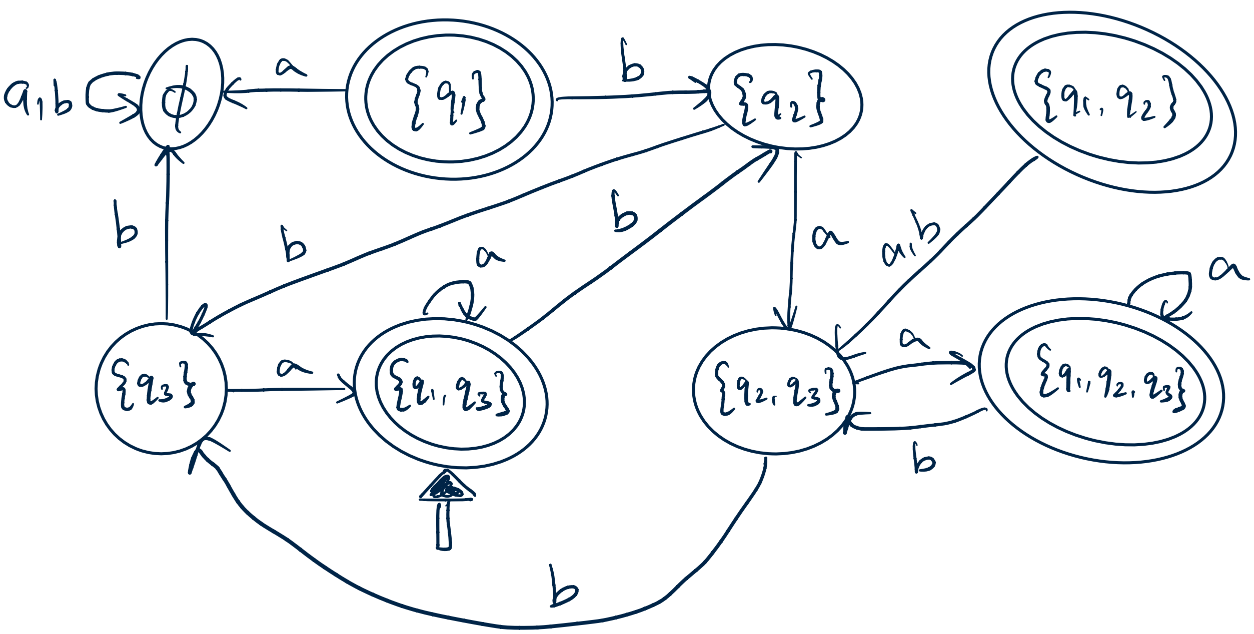
(1) NFA to DFA conversion

NFA
(N) :



$(Q, \Sigma, \delta, q_1, F)$

Equivalent DFA (D): $(Q', \Sigma, \delta', \{q_1, q_3\}, F')$



Start state of N : q_1

Start state of D : $E(\{q_1\}) = \{q_1, q_3\}$

defined in Thm 1.39 (pg 56) Sipser

Accept state(s) of N : $\{q_1\}$

Accept state(s) of D : $\{\{q_1\}, \{q_1, q_2\}, \{q_2, q_3\}, \{q_1, q_2, q_3\}\}$
(i.e F')

Sample computation of transition function:

$\delta'(\{q_1\}, a) = \{\emptyset\}$ because there is no transition defined for (q_1, a) in N.

$\delta'(\{q_3\}, a) = ?$

$\delta(q_3, a) = \{q_1\}$

$E(\{q_1\}) = \boxed{\{q_1, q_3\}}$ ✓

(2) Pumping Lemma for Regular Languages

A is a regular language

$\downarrow$

$\exists p$ (pumping length) such that

$\forall s \in A$, if $|s| \geq p$, then

$\exists x, y, z$ such that

$$s = xyz$$

$$|y| > 0$$

$$|xy| \leq p$$

$$\forall i \geq 0, \quad xy^iz \in A$$

Can be used to prove <u>non</u>-regularity.

i.e A is a regular language $\rightarrow$ $\exists p$ S.T all strings $s \in A$ with $|s| \geq p$ can be pumped.

$\forall p \; \exists s \in A$, $|s| \geq p$, s cannot be pumped $\rightarrow$ A is NOT regular.

<u>Example</u>

Show that $L = REP(\{0^n 1^n \mid n \geq 1\})$ is not regular.

- Let p be an arbitrary positive integer, W.T.S p is not the pumping length for L.

- Let $s = 2 0^p 1^p 2$

- We have :

  a) $s \in L$ because between every pair of successive 2s in s is a string in $\{0^n 1^n ; n \geq 1\}$

  b) $|s| = 2p + 2 > p$ (since p is a +ve integer)

- Consider strings x, y, z such that $s = xyz$, $|y| > 0$, $|xy| \leq p$

Case (1) $x = \epsilon$, $y = 2$, $z = 0^p 1^p 2$

$xyyz = \underbrace{22}0^p 1^p 2 \notin L$

↳ between these 2s is the string $\epsilon \notin \{0^n 1^n \mid n \geq 1\}$

(2) $x = \epsilon$, $y = 20^m$, $z = 0^{p-m} 1^p 2$ $(0 < m < p)$

$xyyz = 20^m 20^m 0^{p-m} 1^p 2 = \underbrace{20^m 2}0^p 1^p 2 \notin L$

↙ between these 2s is the string $0^m \notin \{0^n 1^n \mid n \geq 1\}$

(3) $x = 20^k$, $y = 0^m$, $z = 0^{p-m-k} 1^p 2$ $(k \geq 0, 0 < m < p)$

$xyyz = 20^k 0^m 0^m 0^{p-m-k} 1^p 2$

$= 20^{p+m} 1^p 2 \notin L$ (similar reason)

∴ For any p, we have some counter example s that cannot be pumped with pumping length p. ∴ L is non regular.

(3) (HW6 Q 3b)

For each regular language L, the language $\{\langle M_1, M_2\rangle \mid M_1$ & M_2 are DFAs & $L(M_1) \subseteq L$ & $L(M_2) \subseteq \overline{L}\}$ is decidable. True / False ?

Sol: True.

- Use set identity $X \subseteq Y \Leftrightarrow X \cup Y = Y$
- Since L is regular, there is a DFA (say A) such that $L(A) = L$. Also, since regular languages are closed under complement, there is a DFA (say B) such that $L(B) = \overline{L}$.
- Since EQ_{DFA} is decidable, there is a T.m (say M_{EQ}) that decides EQ_{DFA}.

Define T.m S =

"on input w

1. If w is not a valid encoding $\langle M_1, M_2\rangle$ of 2 DFAs then reject.
2. Construct DFA D_1; $L(D_1) = L(A) \cup L(M_1)$
3. Run M_{EQ} on input $\langle D_1, A\rangle$. If it rejects, reject.
4. Construct DFA D_2; $L(D_2) = L(B) \cup L(M_2)$
5. Run M_{EQ} on input $\langle D_2, B\rangle$. If it rejects, reject.
6. Accept.

(Refer sample solutions for full justification)

(4) Mapping reduction theorems / strategies & sample question.

If $A \le_m B$ then

(a) B is decidable $\rightarrow$ A is decidable

(b) A is undecidable $\rightarrow$ B is undecidable

(c) B is recognizable $\rightarrow$ A is recognizable

(d) A is not recognizable $\rightarrow$ B is not recognizable

We know:

$$\left.\begin{array}{l} \text{(a) } A_{TM} \text{ is recognizable} \\ \text{(b) } \overline{A_{TM}} \text{ is not recognizable} \end{array}\right\} A_{TM} \text{ is undecidable}$$

To prove some language B is NOT recognizable:

Show $\overline{A_{TM}} \le_m B$ (or) $A_{TM} \le_m \overline{B}$

To prove some language B is NOT co-recognizable
(i.e $\overline{B}$ is not recognizable)

Show $\overline{A_{TM}} \le_m \overline{B}$ i.e $A_{TM} \le_m B$

To prove some language B is recognizable you can

a) mapping reduce B to some known recognizable language

or

b) Construct a T.M that recognizes B.

(and similarly for proving B is decidable).

e.g. EQ_{TM} is not recognizable (Thm 5.30 Sipser)

W.T.S $A_{TM} \leq_m \overline{EQ_{TM}}$

F = " On input $\langle M, w \rangle$ where M is a TM, w is a string

1. Construct T.M M_1 = "on input x, reject".
2. Construct T.M M_2 = "On input x
   1. Run M on input w.
   2. If M accepts, accept."
3. Output $\langle M_1, M_2 \rangle$

If M accepts w, M_2 accepts all strings so $L(M_1) \neq L(M_2)$

If M doesn't accept w, M_2 does not accept any string, so $L(M_1) = L(M_2)$

(To show EQ_{TM} is not co-recognizable, change M_1 to "on any input, accept".)

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(5) Proof that $HALT_{TM}$ is undecidable (pg 217 Sipser)

Suppose that $HALT_{TM}$ is decidable. $\exists$ some T.M H that decides it.

Construct T.M S that decides A_{TM} as follows:

S = " on input $\langle M, w \rangle$ where M is a T.M, w is a string

1. Run H on $\langle M, w \rangle$. If H rejects, reject.

2. Simulate M on w. If M accepts, accept.

   If M rejects, reject."

This is a contradiction, since we know A_{TM} is not decidable.
