Disclaimer: Discussion notes represent some of the proper detailed case analysis that goes into writing a formal proof; nevertheless, we still consider these notes to be a "rough guide" to how the proofs should go, and do not necessarily reflect the level of detail and clarity required for homeworks and exams

CSE 105 Discussion Week 1 Friday

| Definitions | Notes | Example |
|---|---|---|
| Symbol is an element of the alphabet. | Symbols are single characters. Thus, $\varepsilon \notin \Sigma$. | a, b, c |
| Alphabet is a non-empty finite set of symbols. | $\Sigma^*$ is the set of all strings over $\Sigma$. | $\Sigma = \{a, b, c\}$ |
| String is a sequence of symbols. | A string over $\Sigma$ has all its symbols in $\Sigma$. | Strings abc, abba, $\varepsilon$ are over $\Sigma$. |
| A language over $\Sigma$ is a set of strings over $\Sigma$. | Language over $\Sigma \subseteq \Sigma^*$ | $\{abc, abba, \varepsilon\}$ is a language over $\Sigma$.<br>$\{an \mid n \geq 0\}$ is a language over $\Sigma$. |
| A regular expression over alphabet $\Sigma$ is a syntactic expression that can describe a language over $\Sigma$. | You can think of regex as sequences of symbols that specify a match pattern, except ø and $\varepsilon$. | $(ab)^*$<br>$(a \cup b)$ |

Regular Expressions :

- Base Step : $\varepsilon$, ø, and a are regular expressions, where a is a symbol in $\Sigma$ .
- Recursive Step : $(R1 \cup R2)$, $(R1 \circ R2)$, $(R1^*)$ are regular expressions, where R1 and R2 are regular expressions.

Language described by regular expression :

$L(\varepsilon) = \{\varepsilon\}$

$L(ø) = ø$

$L(x) = \{x\}$, for any symbol x.

$L(R_1 \cup R_2) = L(R_1) \cup L(R_2) = \{s \mid s \in L(R_1) \text{ or } s \in L(R_2)\}$

$L(R_1 \circ R_2) = L(R_1) \circ L(R_2) = \{uv \mid u \in L(R_1) \text{ and } v \in L(R_2)\}$

$L(R_1^*) = (L(R_1))^* = \{w_1 \dots w_k \mid k \geq 0 \text{ and each } w_i \in L(R_1)\}$

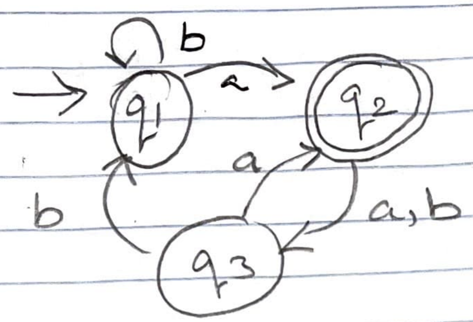
Example :

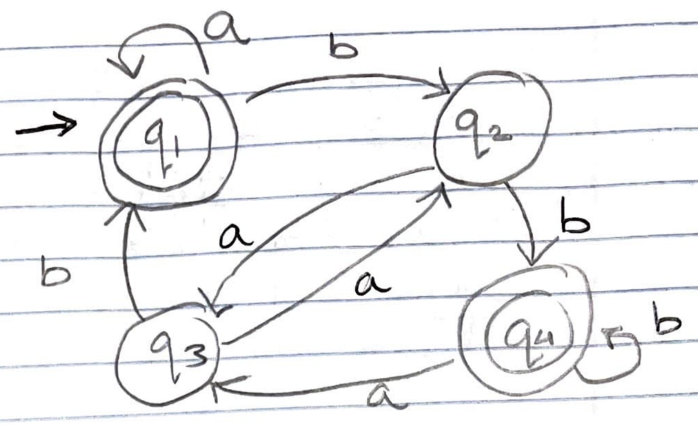
$L(ab^*) = \{a, ab, abb, abbb, \dots \}$

$L((ab)^*) = \{\varepsilon , ab, abab, ababab, \dots \}$

CSE 105 Discussion
April 7, 2023.
Rough Practice Solutions

1.1 $M_1$




(a) Start state has an arrow pointing from nowhere to it.
$M_1 : q_1$ $M_2 : q_1$

(b) Accept states have double circles.
$M_1 : \{q_2\}$ , $M_2 : \{q_1, q_4\}$

(c) Input is aabb
$M_1 : \rightarrow q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_3 \xrightarrow{b} q_1 \xrightarrow{b} q_1$ = Answer
States : $q_1, q_2, q_3, q_1, q_1$

$M_2 : \rightarrow q_1 \xrightarrow{a} q_1 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{b} q_4$ = Answer
States : $q_1, q_1, q_1, q_2, q_4$

(d) The final state on $M_1$ is $q_1$, which is not an accept state. $M_1$ does not accept aabb.

The final state on $M_2$ is $q_4$, which is an accept. $M_2$ does accept aabb.

(e) $M_1$ : No. The final state for empty string $\varepsilon$ is $q_1$, which is not an accept state.
$M_2$ : Yes. The final state for $\varepsilon$ is $q_1$, which is an accept state.

A finite automaton is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where
$Q$ is states, $\Sigma$ is alphabet,
$\delta$ is transition function, $q_0 \in Q$ is the start state and $F \subseteq Q$ is set of accept states.

DFA $\delta : Q \times \Sigma \to Q$
NFA $\delta : Q \times \Sigma_\varepsilon \to P(Q)$,
where $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$,
$P(Q)$ is power set of $Q$.
(Non-determinism will be covered next week)

1.2 $M_1 = (\{q_1, q_2, q_3\}, \{a, b\}, \delta_1, q_1, \{q_2\})$, where

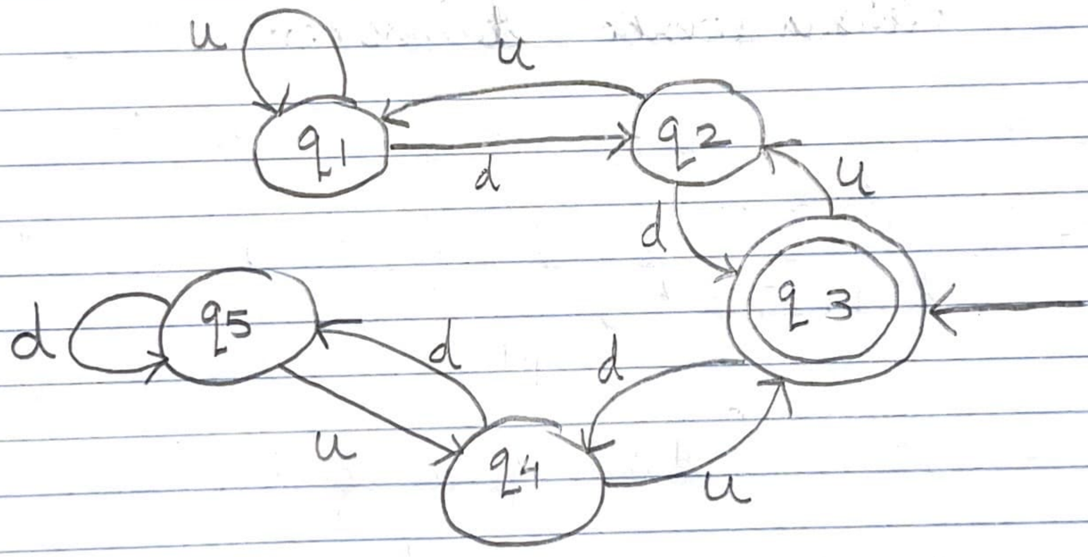
| $\delta_1$ | a | b |
|---|---|---|
| $q_1$ | $q_2$ | $q_1$ |
| $q_2$ | $q_3$ | $q_3$ |
| $q_3$ | $q_2$ | $q_1$ |

$M_2 = (\{q_1, q_2, q_3, q_4\}, \{a, b\}, \delta_2, q_1, \{q_1, q_4\})$

| $\delta_2$ | a | b |
|---|---|---|
| $q_1$ | $q_1$ | $q_2$ |
| $q_2$ | $q_3$ | $q_4$ |
| $q_3$ | $q_2$ | $q_1$ |
| $q_4$ | $q_3$ | $q_4$ |

1.3 $M = (\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_3, \{q_3\})$

| $\delta$ | u | d |
|---|---|---|
| $q_1$ | $q_1$ | $q_2$ |
| $q_2$ | $q_1$ | $q_3$ |
| $q_3$ | $q_2$ | $q_4$ |
| $q_4$ | $q_3$ | $q_5$ |
| $q_5$ | $q_4$ | $q_5$ |




1.18 ⓐ $L_0 = \{w \mid w$ begins with 1 and ends with 0$\}$
$\Sigma = \{0, 1\}$
Let $R = 1\Sigma^* 0$
We want language over regular expression R to be the same as $L_0$.
$L(R) = L_0$
$L(R) = \{10, 100, 110, 1000, \ldots\}$

(b) $R = \Sigma^* 1 \Sigma^* 1 \Sigma^* 1 \Sigma^*$
$L = \{w \mid w$ contains at least three 1s$\}$
(c) $R = \Sigma^* 0101 \Sigma^*$
$L = \{w \mid w$ contains substring $0101\}$
(d) $R = \Sigma \Sigma 0 \Sigma^*$
$L = \{w \mid |w| \geq 3$ and third symbol is $0\}$

(f) $L = \{w \mid w$ doesn't contain the substring $110\}$
$R = 0^* (10^+)^* 1^*$
(Answer changed since discussion section)

(h) $L = \{w \mid w$ is any string except $11$ and $111\}$
$R = (\varepsilon \cup 1 \cup 0\Sigma^* \cup 10\Sigma^* \cup 110\Sigma^* \cup 111\Sigma\Sigma^*)$

1.23 $B^+ = BB^*$ (Solution given back of the chapter)

Case 1 : Assume $B = B^+$
$B^+ = BB^*$
$\Rightarrow BB \subseteq BB^* = B^+ = B$
$\Rightarrow BB \subseteq B$

$\therefore B = B^+ \Rightarrow BB \subseteq B$

Case 2 : Assume $BB \subseteq B$
$B \subseteq BB^* = B^+$
$\Rightarrow B \subseteq B^+$

Suppose $w \in B^+$.
$\exists\, x_1, x_2, \ldots, x_k \in B$ such that
$w = x_1 x_2 \cdots x_k$, for some $k \geq 1$.

$x_1, x_2 \in B$
$\Rightarrow x_1 x_2 \in BB$
$\Rightarrow x_1 x_2 \in B$

$x_1 x_2, \; x_3 \in B$
$\Rightarrow x_1 x_2 x_3 \in BB$
$\Rightarrow x_1 x_2 x_3 \in B$

...

$\Rightarrow w = x_1 x_2 \cdots x_k \in B$

Thus, $\forall w \in B^+$, $w \in B$
$\Rightarrow B^+ \subseteq B$

$B^+ \subseteq B$ and $B \subseteq B^+$
$\Rightarrow B^+ = B$

Therefore, $BB \subseteq B \Rightarrow B^+ = B$

Proved. $BB \subseteq B$ iff $B^+ = B$