CSE 105 Spring 2023
Week 3 Discussion

Pumping Lemma : If A is a regular language, then there exists a number p such that for any string s in A of length at least p, s can be divided into 3 parts xyz, such that

- $|y| > 0$
- $|xy| \leq p$
- $xy^iz \in A, \forall\ i \geq 0$

Using pumping lemma to prove that a language B is non-regular : Let p be the pumping length given by the pumping lemma for language B. There exists string s in B of length at least p, such that $\forall$ x,y,z such that s = xyz, $|xy| \leq p$ and $|y| > 0$, $\exists\ i \geq 0$ such that $xy^iz \notin B$.

Question 1.29a :
Let p be the pumping length given by the pumping lemma for language $A_1$.
Let $s = 0^p1^p2^p$.
$s \in A_1$ and $|s| \geq p$.
Let b > 0, a such that $a+b \leq p$, $x = 0^a$, $y = 0^b$, $z = 0^{p-a-b}1^p2^p$
$|xy| = a+b \leq p$, $|y| = b > 0$.
$xy^0z = 0^a0^{p-a-b}1^p2^p = 0^{p-b}1^p2^p$.
Since $p-b \neq p$, $0^{p-b}1^p2^p \notin A_1$.
$\forall$ x,y,z such that s = xyz, $|xy| \leq p$ and $|y| > 0$, $xy^0z \notin A_1$.
$A_1$ is non-regular.

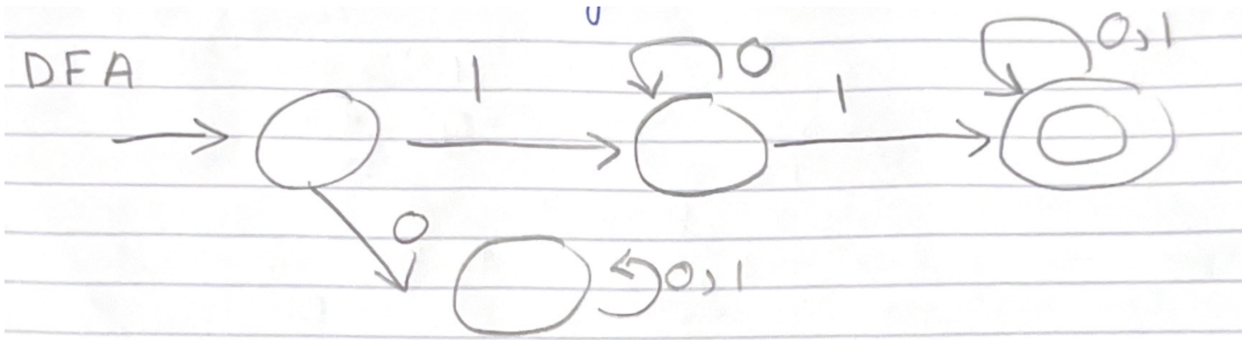
Question 1.29b :
$A_2 = \{www \mid w \in \{a,b\}^*\}$
Let p be the pumping length given by the pumping lemma for language $A_2$.
Let $s = a^pba^pba^pb$.
$s \in A_2$ and $|s| \geq p$.
Let n > 0, m such that $m+n \leq p$, $x = a^m$, $y = a^n$, $z = a^{p-n-m}ba^pba^pb$
$|xy| = m+n \leq p$, $|y| = n > 0$.
$xy^0z = a^ma^{p-n-m}ba^pba^pb = a^{p-n}ba^pba^pb$
$a^{p-n}ba^pba^pb \notin A_2$.
$\forall$ x,y,z such that s = xyz, $|xy| \leq p$ and $|y| > 0$, $xy^0z \notin A_2$.
$A_2$ is non-regular.

Question 1.29c : Answer at back of the book.

Question 1.30 : Language in example 1.73 is $L(\{0^n1^n \mid n \geq 0\})$. We cannot use the same prove for language L(0*1*).

Question 1.49 a : Prove B is regular.
$B = \{1^ky \mid y \in \{0,1\}^*$ and y contains at least k 1s, for $k \geq 1\}$

A string in B must start with 1 and have another 1.




Question 1.49 b : Prove C is nonregular.
C = $\{1^k y \mid y \in \{0,1\}^*$ and y contains at most k 1s, for $k \geq 1\}$

Let p be the pumping length given by the pumping lemma for language C.
Let t = $01^p$
Let s = $1^p t$.
$s \in C$, $|s| \geq p$.

Let n > 0, m such that $m+n \leq p$.
Let $x = 1^m$, $y = 1^n$, $z = 1^{p-m-n}01^p$
$xy^0z = 1^{p-n}01^p$.
$n < 0 \Rightarrow p-n < p$.
$\Rightarrow xy^0z \notin C$.

Thus, $\forall$ x, y, z such that s = xyz, $|y| > 0$, $|xy| \leq p$, $xy^0z \notin C$.
C is nonregular.

Question 1.50 : Find the minimum pumping length.

Part a : Answer at the back of the chapter

Part b : Answer at the back of the chapter

Part c : $001 \cup 0^*1^*$
$001 \in 0^*1^*$
$L(001 \cup 0^*1^*) = L(0^*1^*)$
Minimum pumping length is 1. Answer of part c.

Part d : Answer at the back of the chapter

Part e : $(01)^*$

Minimum pumping length is 2. Any non-empty string of L((01)*) can be partitioned into $x = \varepsilon$, $y = (01)$, z = remainder of string.

Minimum pumping length cannot be 1 because |xy| must be less than p and |y| > 0.
The only choice for x, y, z would be $\varepsilon$, 0, and remainder of the string respectively.
z would start with 1 and $z \notin (01)^*$
$xy^0z = \varepsilon\, \varepsilon\, z = z \notin (01)^*$
Thus, p cannot be 1.

Part f : $\varepsilon$
Pumping length must be greater than 0.
Minimum pumping length is 1.

Part g : G = 1*01*01*
$00 \in 1^*01^*01^*$, but cannot be pumped.

Assume p = 2.
Let n > 0, m such that $m+n \leq p$.
Consider string s = 00.
Let $x = 0^m$, $y = 0^n$, $z = 0^{2-m-n}$.
$xy^0z = 0^m\, \varepsilon\, 0^{2-m-n} = 0^{2-n}$
$n \geq 1$
$2-n \leq 1$.
$\Rightarrow xy^0z = 0 \notin G$ or $xy^0z = \varepsilon \notin G$
⇒ Contradiction
⇒ p cannot be 2.

Part h : 10(11*0)*0
Minimum pumping length is 4.
$p \neq 3$ because 100 cannot be pumped, for any y, for any i.

Part i : 1011
Minimum pumping length is 5. 1011 cannot be pumped.

Part j : Σ*
Minimum pumping length is 1.
We can partition any string $s \in \Sigma^*$ into $x = \varepsilon$, y = s, and $z = \varepsilon$.
$y \in \Sigma^*$
$y^i \in \Sigma^*,\ \forall\ i \geq 0$.
$xy^iz \in \Sigma^*,\ \forall\ i \geq 0$.