

Context-Free Grammars

$$G_1 = (\underbrace{\{X, Y, Z\}}_{\text{variables}}, \underbrace{\{0, 1, 2\}}_{\text{terminals}}, \underbrace{R}_{\text{rules}}, \underbrace{X}_{\text{start variable}})$$

$R =$ X can be replaced with either $2X2$ or Y

$$X \rightarrow 2X2 \mid Y$$

$$X \Rightarrow 2X2 \Rightarrow 2Y2 \Rightarrow 200Y112 \Rightarrow 200112 \leftarrow \text{in } L(G_1)$$

$$Y \rightarrow 00Y11 \mid \epsilon$$

$$22001122$$

$$X \Rightarrow 2X2 \Rightarrow 22X22 \Rightarrow 22Y22 \Rightarrow 2200Y1122 \Rightarrow 22001122 \checkmark \text{ in } L(G_1)$$

$$\boxed{Z \rightarrow 1X} \leftarrow \text{unreachable when } X \text{ is the start variable}$$

$$L(G_1) = \{2^n (00)^k (11)^k 2^n \mid n \geq 0, k \geq 0\} - \{2^n 0^m 1^m 2^n \mid n \geq 0, m \geq 0, m \text{ is even}\}$$

If we change the start variable to Z : $L(G_1') = (\{X, Y, Z\}, \{0, 1, 2\}, R, Z)$

$$L(G_1') = \{1 2^n (00)^k (11)^k 2^n \mid n \geq 0, k \geq 0\}$$

$$G_2 = (\{S, T\}, \{0, 1, 2\}, R, S)$$

that is what we want
desire
correct

R = BAD WRONG
 $S \rightarrow 0S1 \mid 1T \mid T0$
 $T \rightarrow 1T \mid 0T$

← Generates the language with all strings over $\{0, 1\}$ except for $\{0^n 1^n \mid n \geq 0\}$???
 do not use

NO Any string that contains the symbol 2 is not in the language.

Doesn't allow us to terminate the derivation!
 Generates \emptyset in this case.

In the language: 10 006
 Not in the language: 01 000111 ϵ

CORRECT 😊

R =
 $S \rightarrow 0S1 \mid 1T \mid T0$
 $T \rightarrow 1T \mid 0T \mid \underline{\epsilon}$

YIPPEE

GOOD YAY BEHAV

01 ~~S~~ \Rightarrow 0S1 ~~S~~ \Rightarrow 00S11 \Rightarrow 000S111 ~~S~~

1000
 $S \Rightarrow 1T \Rightarrow 10T \Rightarrow 100T \Rightarrow 1000T$

We need to stop ~~1111~~, but the rules don't allow us to!

Design a CFG to generate the set of strings over $\{0, 1, 2\}$ where
 (the number of 0's + the number of 1's) \geq (the number of 2's)

$$G_3 = (\{J, K, L\}, \{0, 1, 2\}, R, J)$$

R =

$$J \rightarrow K \mid KJ \mid L$$

$$K \rightarrow \epsilon \mid 2KL \mid LK2$$

$$L \rightarrow 0 \mid 1 \mid LL$$

$$22110$$

$$J \Rightarrow K \Rightarrow 2KL \Rightarrow 22KLL \Rightarrow 22LL$$

$$\Rightarrow 221L \Rightarrow 2211$$

20102

~~$$K \Rightarrow 2KL \Rightarrow 2LK2L \Rightarrow$$~~
~~$$2L2L \Rightarrow 202L \Rightarrow \emptyset$$~~

$$J \Rightarrow KJ \Rightarrow 2KLJ \Rightarrow 2LJ \Rightarrow 20J$$

$$\Rightarrow 20K \Rightarrow 20LK2 \Rightarrow 20L2$$

$$\Rightarrow 2002 \quad \checkmark$$

G₁₂₃

Design a CFG to generate the language $L(G_1) \cup L(G_2) \cup L(G_3)$

$G_1 = (\{X, Y, Z\}, \{0, 1, 2\}, R_1, X)$

$R_1 =$
 $X \rightarrow 2X2 \mid Y$
 $Y \rightarrow 00Y11 \mid \epsilon$
 $Z \rightarrow 1X$

$G_2 = (\{S, T\}, \{0, 1, 2\}, R_2, S)$

$R_2 =$
 $S \rightarrow 0S1 \mid 1T \mid T0$
 $T \rightarrow 1T \mid 0T \mid \epsilon$

$G_3 = (\{J, K, L\}, \{0, 1, 2\}, R_3, J)$

$R_3 =$
 $J \rightarrow K \mid KJ \mid L$
 $K \rightarrow 2KL \mid LK2 \mid \epsilon$
 $L \rightarrow 0 \mid 1 \mid LL$

$G_{123} = (\{X, Y, Z, S, T, J, K, L\}, \{0, 1, 2\}, R_{123}, A)$

$R_{123} =$

$A \rightarrow X \mid S \mid J$

$X \rightarrow 2X2 \mid Y$
 $Y \rightarrow 00Y11 \mid \epsilon$
 $Z \rightarrow 1X$

$S \rightarrow 0S1 \mid 1T \mid T0$
 $T \rightarrow 1T \mid 0T \mid \epsilon$

$J \rightarrow K \mid KJ \mid L$
 $K \rightarrow 2KL \mid LK2 \mid \epsilon$
 $L \rightarrow 0 \mid 1 \mid LL$